

# Skyrmions and Bags in the $2D$ - $O(3)$ model

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## Abstract

Localized static solutions of the  $2D$ - $O(3)$  model are investigated in a representation with the 3-vector field  $\Phi$  split into the unit vector  $\hat{\Phi}$  and the modulus  $\Phi$ . As in the nonlinear version of the model this allows for the definition of a topological winding number  $B$ , and for the separation of the complete configuration space into distinct  $B$ -sectors. For small values of the  $\Phi^4$ -coupling strength the stable energy minima in these sectors are characterized by bag formation in the modulus field which in the standard cartesian representation of the linear  $O(3)$  model would be unstable towards decay into the trivial  $B = 0$  vacuum. Stabilized by  $B$ -conservation they exhibit a surprising variety of very appealing features for multiply charged systems. With the total charge bound into one common deep bag opposite ways of distributing the topological charge density inside the bag can be realized: Pointlike structures which retain the individuality of single constituents (or doubly charged pairs), or a deconfined charge density spread uniformly throughout the interior of the bag. It is suggested that this extension supplies a crucial link to overcome the unsatisfactory existing mismatch between multiskyrmion configurations and nuclear structure.

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## 1. Introduction

The linear  $2D-O(3)$  model has been a favorite tool as a model field theory for a wide spectrum of physical systems. Commonly, the field vector  $\Phi$  is embedded into a euclidean manifold and parametrized in terms of three cartesian components  $\Phi_i$ , ( $i=1,2,3$ ). Topologically this manifold is trivially connected. For standard  $\Phi^4$  potentials of the type  $(\sum_i \Phi_i^2 - 1)^2$  the vacuum manifold is the 2-sphere  $S^2$ . Addition of symmetry breakers can remove the degeneracy of the vacuum manifold. Imposing the constraint  $\sum_i \Phi_i^2 \equiv 1$  defines the nonlinear  $2D-O(3)$  model. The homotopy  $S^2 \rightarrow S^2$  then allows for a classification of static solutions by an integer topological winding index  $B$  which is the spatial integral over the time component of a conserved topological current. Nontrivial configurations with  $B \neq 0$  have found a direct application for the interpretation of the Quantum Hall Effect as charged excitations in two-dimensional spin systems [1]. In the linear model, where configurations are not confined to the 2-sphere  $S^2$ , the winding number  $B$  is not topologically protected. Therefore, beyond a critical magnitude for the coupling constants of symmetry breakers, static solutions with non vanishing winding  $B \neq 0$  become unstable, and they unwind into the topologically trivial ( $B = 0$ ) uniform vacuum. In time-dependent evolutions changes in  $B$  will frequently occur whenever the field passes through  $\Phi = 0$  at some point  $\mathbf{x}, t$  in space and time.

In certain applications of the model the length of the  $\Phi$ -vector is an important degree of freedom such that the constraint to  $S^2$  is too restrictive, but still, the winding number may correspond to a physically observable conserved charge. A prominent example is the chiral phase transition where the order parameter is the chiral meson field, its length measures the amount of spontaneous symmetry breaking, while the Skyrme-Witten conjecture [2] identifies the winding index with baryon number. Its conservation law should not be affected by the chiral transition. Then, in order to retain the model as a faithful image of the underlying physics, it is necessary to prevent unwinding of nontrivial configurations. To achieve this it is sufficient to exclude one point from the manifold on which the fields live. The natural choice is to remove the origin  $\Phi = 0$ , which is automatically accomplished in the radial-angular representation  $\Phi = \Phi \hat{\Phi}$  because the angular fields are not defined at  $\Phi = 0$ . Then the manifold is nontrivially connected and the winding number is topologically protected. Thus, choosing the appropriate embedding for the field is an important part of the definition of the model.

In lattice simulations the topological arguments based on continuity cannot be used because field configurations may differ arbitrarily between two neighbouring lattice points. However, the conservation law is easily implemented into the update-algorithm, by allowing only for configurations with specified value of  $B$ . Evidently, for that purpose it is necessary to be able to evaluate the winding number for each configuration, so it is necessary to work in the representation  $\Phi = \Phi \hat{\Phi}$ , where the winding density is defined in terms of angles which may vary arbitrarily from one lattice point to its neighbours.

So, in the following, we shall be dealing with all degrees of freedom of the linear  $O(3)$  model, however in the form of a nonlinear  $O(3)$  model supplemented by an additional modulus field  $\Phi$ . In this framework it is possible to define and conserve the winding number of nontrivial configurations. Naturally, in a sector with specified  $B \neq 0$ , we will

find static solutions which do not exist in the topologically trivial cartesian embedding of the linear  $O(3)$  model and which are characterized by the formation of a localized spatial bag in the modulus field.

We shall discuss two versions of the model which differ in the definition of the current that enters the current-current coupling Skyrme term. The competition between a symmetry-breaking Zeeman term and the Skyrme term determines the spatial extent of the soliton solutions. Thus the two different versions will lead to a characteristic difference in the spatial structure of the resulting winding density distributions in the interior of the bags: A dominating Skyrme term deconfines the topological charge inside the bag while a dominating Zeeman term preserves the particle nature of individual charge units.

## 2. The $2D$ - $O(3)$ model with Skyrme-Zeeman stabilization

We consider the  $O(3)$  lagrangian density in  $2 + 1$  dimensions in terms of the dimensionless 3-component field  $\Phi = \Phi \hat{\Phi}$  with  $\hat{\Phi} \cdot \hat{\Phi} = 1$ ,

$$\mathcal{L} = F^2 \left( \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4\ell^2} (\Phi^2 - f^2)^2 - \frac{\alpha}{\ell^2} (f_0 - \Phi_3) - (\alpha \ell^2) \rho_\mu \rho^\mu \right). \quad (1)$$

Apart from the usual kinetic term this lagrangian contains the standard  $\Phi^4$  potential for the modulus field  $\Phi$  to monitor the spontaneous symmetry breaking with (dimensionless) coupling parameter  $\lambda$ , an explicitly symmetry-breaking ('Zeeman') coupling with (dimensionless) strength  $\alpha$ , and a four-derivative ('Skyrme') current-current coupling  $\rho_\mu \rho^\mu$  for the conserved topological current

$$\rho^\mu = \frac{1}{8\pi} \epsilon^{\mu\nu\rho} \hat{\Phi} \cdot (\partial_\nu \hat{\Phi} \times \partial_\rho \hat{\Phi}), \quad (2)$$

satisfying  $\partial_\mu \rho^\mu = 0$ . The parameter  $F^2$  sets the overall energy scale, the length  $\ell$  may be absorbed into the space-time coordinates, so it sets the size of localized static solutions. In order to keep the uniform minimum of the potential for finite  $\alpha$  at the vacuum value  $\Phi = f_0$  we define

$$f^2 = f_0^2 - \frac{\alpha}{\lambda f_0}, \quad (3)$$

then we have the  $\alpha$ -independent boundary condition for the modulus  $\Phi$  at spatial infinity  $\Phi \rightarrow f_0$  for  $r \rightarrow \infty$ . Of course, we are free to insert additional powers of the modulus field  $\Phi$  into the Skyrme and the Zeeman term, the above choice being motivated to minimize interference with the  $\Phi^4$  spontaneous symmetry-breaking mechanism. This choice implies that as  $f_0$  goes to zero (e.g. with increasing temperature) the typical size  $\ell$  of static defects grows like  $f_0^{-1/4}$ . This may be physically not unreasonable (cf. e.g. the discussion in the 3-dimensional case in [3]). (We shall discuss a different natural choice in the next section). Having fixed the  $\Phi$ -dependence of the lagrangian as given in (1) and (2) we conveniently redefine the field and the parameters by

$$\tilde{\Phi} = \Phi f_0^{-1}, \quad \tilde{F}^2 = F^2 f_0^2, \quad \tilde{\ell} = \ell f_0^{-1/4}, \quad (4)$$

$$\tilde{\lambda} = \lambda f_0^{3/2}, \quad \tilde{\alpha} = \alpha f_0^{-3/2}, \quad \tilde{f}^2 = 1 - \frac{\tilde{\alpha}}{\tilde{\lambda}}, \quad (5)$$

omit the tildes in the following and absorb the  $\ell$ 's into the length scale of space-time. Then we finally have

$$\mathcal{L} = F^2 \left( \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4} \left( \Phi^2 - 1 + \frac{\alpha}{\lambda} \right)^2 - \alpha \rho_\mu \rho^\mu - \alpha (1 - \Phi_3) + \frac{\alpha^2}{4\lambda} \right). \quad (6)$$

together with the  $\alpha$ -independent boundary condition  $\Phi \rightarrow 1$  for  $r \rightarrow \infty$ . The constant  $\alpha^2/(4\lambda)$  has been added to set the  $\Phi$ -potential to zero at the minimum  $\Phi = 1$ . The *nonlinear*  $O(3)$ -model emerges in the limit  $\lambda \rightarrow \infty$  where  $\Phi$  is confined to the 2-sphere  $\Phi^2 \equiv 1$ . For this case the static solutions for the angular field  $\hat{\Phi}$  have been thoroughly investigated [4]. They are characterized by fixed integer winding number  $B$ , and by a modulus field  $\Phi(\mathbf{x})$  which for very large  $\lambda$  differs only minimally from its vacuum value  $\Phi = 1$ . We shall in the following denote them as  $B$ -skyrmions.

For fixed  $\alpha \neq 0$  and sufficiently small values of  $\lambda$  the point  $\Phi = 1$  is the only real minimum of the potential in (6), i.e. for  $\lambda$  smaller than some critical value  $\lambda_c$  static solutions in the cartesian representation of the *linear*  $O(3)$ -model collapse to the trivial vacuum  $\Phi \equiv 1$ . In a  $\lambda$ - $\alpha$  diagram the line  $\lambda_c(\alpha)$  separates two phase regions where for  $\lambda > \lambda_c(\alpha)$  static multi-skyrmions with winding number  $B$  can exist at local energy minima  $E_B$ , while for  $\lambda < \lambda_c(\alpha)$  only the global minimum  $\Phi \equiv 1$  at  $E_0 = 0$  survives. There will be different phase boundaries  $\lambda_c(\alpha)$  for multi-skyrmions with different winding numbers  $B$ .

In the  $\Phi = \Phi \hat{\Phi}$  representation of the linear  $O(3)$  model the winding number is fixed (and can be held fixed in lattice simulations). This means that for  $\lambda > \lambda_c(\alpha)$  the local (multi-skyrmion) minimum  $E_B$  turns into a global minimum in the respective  $B$ -sector and for  $\lambda < \lambda_c(\alpha)$  the collapse of the multi-skyrmions into the vacuum is prevented. Instead, the global minimum in a given  $B$ -sector continues to exist for decreasing values of  $\lambda$ . But we expect a qualitative change in the structure of stable static configurations of (6) in the vicinity of the phase boundaries  $\lambda_c(\alpha)$  leading to new types of solutions for  $\lambda < \lambda_c$  which do not exist in the cartesian embedding of the linear  $O(3)$ -model. The nature of this structural change is easily visualized: Destabilization and unwinding can only proceed through the field configuration passing through  $\Phi = 0$  at some point. Conservation of winding number therefore leads to formation of a spatial bag: In its interior the modulus field  $\Phi$  is close to zero, with rapid variation of the angular fields  $\hat{\Phi}$  such that the winding density is located inside the bag. Thus we may expect a close relation between the deviation of the scalar modulus field from its background value  $\Phi = 1$  and the density of the topological charge. We shall in the following denote these types of solutions as  $B$ -bags.

### 3. Bag formation

Within a given  $B$ -sector the transition from skyrmions to bags is smooth. As  $\lambda$  approaches  $\lambda_c$  from above creation of the bag begins through formation of a shallow depression in the modulus  $\Phi$  near the center of the skyrmions. As  $\lambda$  passes the critical value the depression quickly deepens with the winding density accumulating inside. In fig. 1 this transition is illustrated in the  $B = 1$  and  $B = 5$  sectors. Plotted is the minimal energy per unit topological charge  $E_B(\alpha, \lambda)/B$  as function of  $\lambda$  for three different values of  $\alpha$ . The full lines show the  $B = 1$  sector. For a small value of  $\alpha = 0.1$  the transition near  $\log \lambda \approx 3.2$  is most pronounced. Bag formation sets in rather sharply, with a sudden steep decrease in  $E_B$  with decreasing  $\lambda$ . For very large  $\lambda$  above this critical value,  $E_B$  rises very slowly towards a limit slightly above the Belavin-Polyakov (BP) monopole energy of  $4\pi$  [5]. For larger values of  $\alpha$  the transition region gets smoothed out, still with a large gain in energy through formation of the bag.

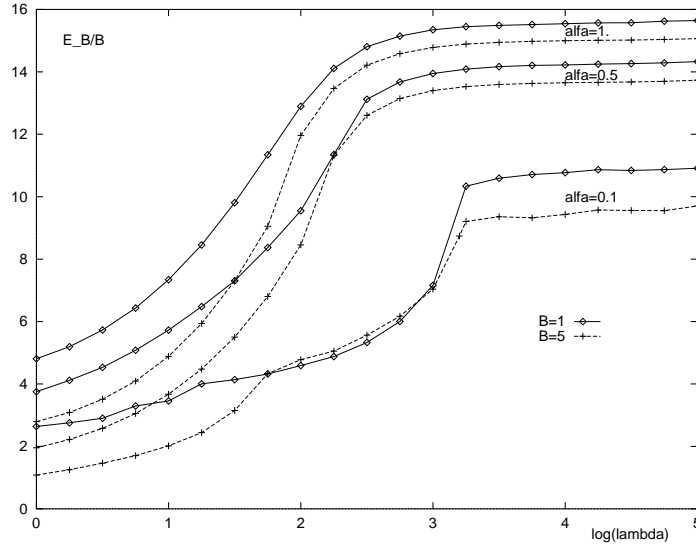


FIG. 1. Minimal energies per unit topological charge  $E_B(\alpha, \lambda)/B$  as function of  $\log \lambda$  for three different values of  $\alpha = 0.1, 0.5, 1.0$ . Full and dashed lines show the cases  $B = 1$  and  $B = 5$ , respectively.

The dashed lines show the same features in the  $B = 5$  sector. Comparing the energies for  $B = 1$  and  $B = 5$  for small  $\lambda$  values near  $\lambda \approx 1$  which correspond to well-developed deep bags we see that  $E_5 < 5E_1$  for all three values of  $\alpha$ . This implies that the five topological charges are strongly bound in the compact  $B = 5$  bag. It is only for the smallest value of  $\alpha$  considered that with increasing  $\lambda$  near  $\log \lambda \approx 1.75$  the 5-bag breaks up into five individual 1-bags such that  $E_5/5$  closely follows  $E_1$  into the transition region where the bags disappear and the emerging five 1-skyrmions combine into pairs of two 2-skyrmions plus one left-over 1-skyrmion (for  $\log \lambda > 3.2$ ). For the larger  $\alpha$ -values considered,  $E_5/5$  always stays below  $E_1$ , so at no point there is a complete breakup into five individual 1-bags.

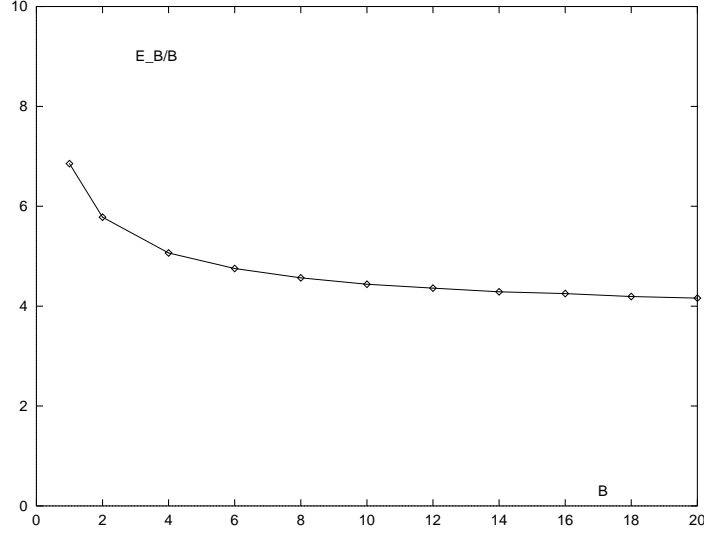


FIG. 2. Minimal energy  $E_B$  per unit topological charge  $B$  as function of  $B$ . ( $\lambda = 10$ ,  $\alpha = 1$ ).

This holds for all values of  $B$  for sufficiently small  $\lambda$ . Figure 2 shows the energy per unit topological charge at  $\lambda = 10$  and  $\alpha = 1$ . It is a monotonically decreasing function of  $B$  which converges towards approximately one half of the  $B = 1$  energy; so for this  $\lambda$  all B-bags are stable against decay into 1-bags. Altogether there emerges a phase diagram of remarkable richness. The skyrmion regions ( $\lambda \gg 1$ ) have been amply discussed, so here we only illustrate typical features of the region with well developed deep bags. (In order to get sufficient resolution the results are calculated for  $\ell = 10$  on an  $80 \times 80$  mesh.)

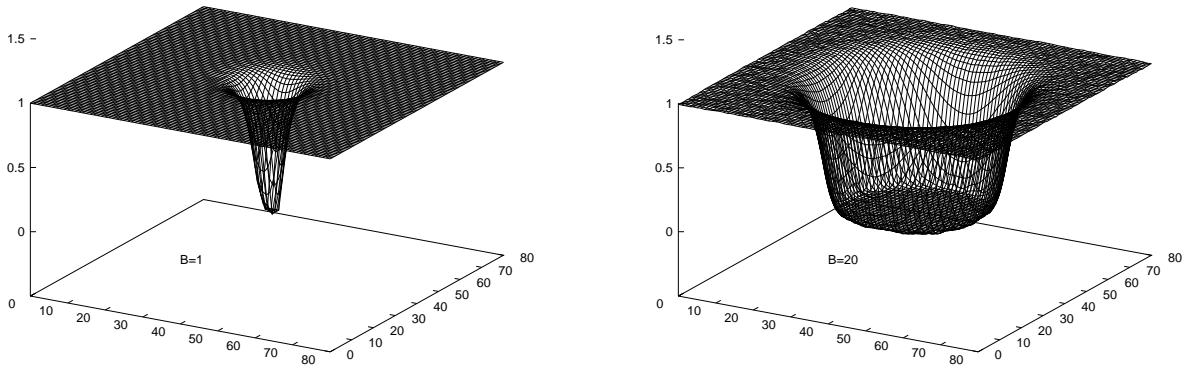


FIG. 3. 3D-view of the bags formed for  $B = 1$  (a), and  $B = 20$  (b), (for  $\alpha = 1$ ,  $\lambda = 10$ ).

The 1-bag which is formed for  $\alpha = 1$  and  $\lambda = 10$  is shown in fig. 3a. Its total classical energy is  $E_1 = 6.85$ . Already for this smallest value of  $B$  the bag has developed a basically flat bottom where the modulus field is very close to zero, with small numerical

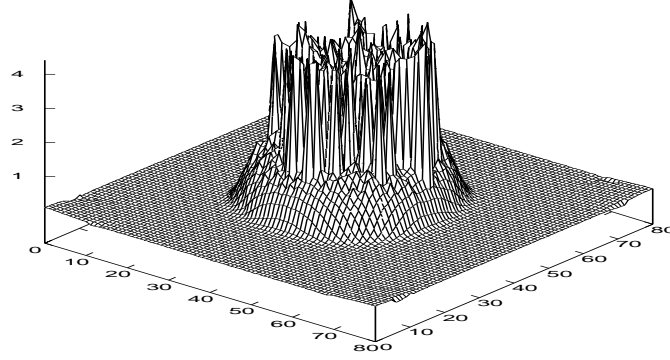


FIG. 4. 3D-view of the angular field  $\Theta(\mathbf{x}) = \arccos \hat{\Phi}_3(\mathbf{x})$  for  $B = 20$  ( $\lambda = 10$ ,  $\alpha = 1$ ), fluctuating rapidly inside the bag of fig. 3b around an average value of  $\pi$ .

fluctuations. To accommodate increasing numbers of topological charge  $B$  the bag size increases correspondingly, its depth being limited by  $\Phi > 0$ . As an example, fig.3b shows the case  $B = 20$  for  $\lambda = 10$  and  $\alpha = 1$ . The near-central cuts through the bag profiles plotted in fig. 5a (for  $B=1,2,4,6,\dots,20$ ) show that the surface thickness of the bags is basically independent of  $B$ , it is only the radius of the flat interior which adjusts to the increasing total charge.

Throughout this deep bottom of the bag the angular fields display numerous strong local fluctuations. (Fig.4 shows the angle  $\Theta = \arccos \hat{\Phi}_3$  (for  $B = 20$ ) fluctuating rapidly inside the bag around an average value of  $\pi$ ). Such fluctuations are energetically harmless because the large gradient terms are multiplied by the almost vanishing square of the modulus field. By this mechanism an essentially constant winding density  $\rho_0$  is achieved which extends over the whole flat interior of the bag (apart from small numerical fluctuations, see fig.5b). In the context of nuclear physics one would say that the nucleons have lost their identity inside the nucleus and have dissolved into a hadronic soup. In fig.5b the near-central cuts through the density profiles are shown which correspond to the same bags as given in fig.5a. Again the surface thickness of the profiles is basically constant; it is, however, smaller than that of the bag profiles. So, for larger values of  $B$  a square slab closely approximates the density distributions. This is to be contrasted with the winding density of a B-skyrmion. Its angular fields closely resemble the BP monopole  $\Theta(r) = 2 \arctan(B/r)$  with winding density

$$\rho_{BP} = -\frac{B\Theta' \sin \Theta}{4\pi r} = \frac{B^3}{\pi(r^2 + B^2)^2}. \quad (7)$$

There is no central plateau, the center density decreases as  $\rho_{BP}(0) \propto B^{-1}$  and the mean radius increases as  $R \propto B$ . In contrast to this, fig.5b shows that for the B-bags the central density  $\rho_0$  quickly drops from its maximal value for  $B = 1$  ( $\rho_0 = 0.025$  for  $\lambda = 10$ ,  $\alpha = 1$ ), then slowly converges with increasing  $B$  towards a constant value, which for this parameter

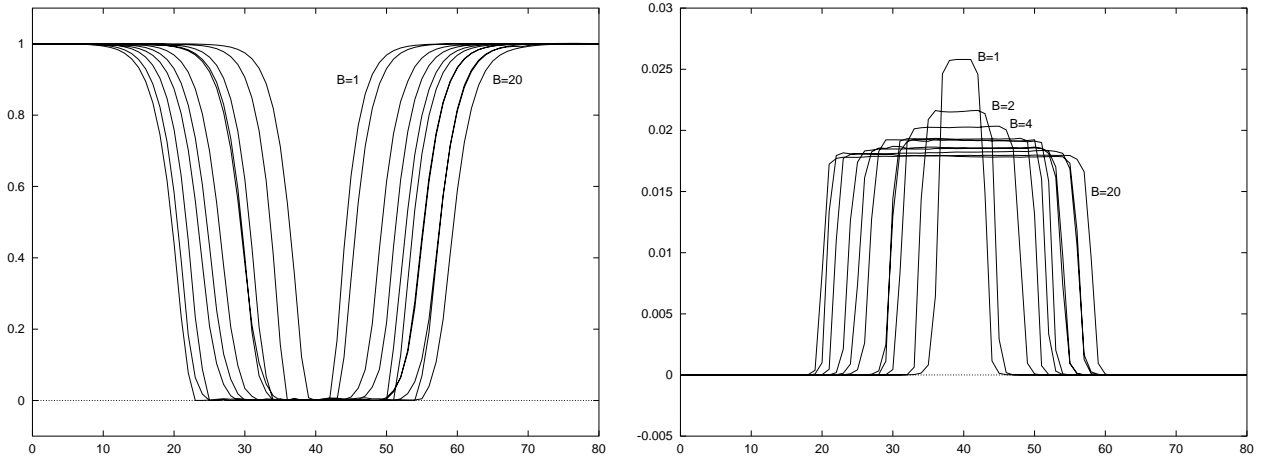


FIG. 5. Near-central cuts through the bag profiles (a), and the corresponding winding density profiles (b), (for  $B=1,2,4,6,\dots,20$ ) for  $\lambda = 10$ ,  $\alpha = 1$ . (Due to numerical fluctuations the positions of the bag centers may differ slightly for different values of  $B$ .)

set is near  $\rho_0 \approx 0.017$ . This  $B$ -dependence reflects the interplay between the surface energy of the bag (which originates in the gradient terms of the modulus field) and volume energies which comprise the potential energy of the modulus field, Zeeman and Skyrme terms, and kinetic terms due to the gradients of the angular fields in the linear  $\sigma$ -model part of the lagrangian (6).

To get an idea about the ('nuclear matter') density  $\rho_0$  in the limit  $B \rightarrow \infty$  (where surface terms play no role) we could approximately replace the winding density defined in terms of the angular fields (2) by the deviation of the modulus field  $\Phi$  from its vacuum value

$$\rho \approx \rho_0(1 - \Phi) \quad (8)$$

and obtain the bag radius  $R$  from  $\pi R^2 \rho_0 = B$ . The lagrangian (1) then shows that creation of a deep bag ( $\Phi \approx 0$  inside,  $\Phi=1$  outside) requires a volume potential energy density of

$$\epsilon_V = \frac{\lambda + 2\alpha}{4\ell^2} + \ell^2 \alpha \rho_0^2. \quad (9)$$

Ignoring kinetic contributions for the moment, for fixed  $B$  the interplay between these two volume terms determines the average density  $\rho_0$  as

$$\rho_0 = \frac{1}{2\ell^2} \sqrt{\frac{\lambda + 2\alpha}{\alpha}}. \quad (10)$$

which for  $\lambda = 10$  and  $\alpha = 1$  is  $\rho_0 = \sqrt{3}/\ell^2 = 0.0173$ .

The additional pressure due to the kinetic volume terms will further lower the central density, but comparison with the numerical result in fig.5b shows that their influence



must be small. This is what we might expect due to the smallness of  $\Phi$  inside the bag, although very large gradients in  $\hat{\Phi}$  could compensate for it and cause a noticeable lowering of  $\rho_0$ , especially for small values of  $\alpha$ . Maximal importance of kinetic terms would occur if the hadronic soup inside the bag would consist of fermions. Then, in Thomas-Fermi approximation for the density  $\rho_0 = p_F^2/(4\pi)$  we would have for the energy per particle

$$\frac{\epsilon}{\rho_0} = \frac{4\sqrt{\pi}}{3} \sqrt{\ell^2 \rho_0} + \frac{\epsilon_V}{\rho_0}. \quad (11)$$

Variation with respect to  $\rho_0$  then leads to

$$\frac{2\sqrt{\pi}}{3} \ell \rho_0^{3/2} - \frac{\lambda + 2\alpha}{4\ell^2} + \ell^2 \alpha \rho_0^2 = 0. \quad (12)$$

For  $\lambda = 10$  and  $\alpha = 1$  the central density thereby is lowered from 0.0173 to  $\rho_0 = 0.0120$ . Comparison with fig.5b apparently rules out the conjecture of a fermionic soup. (For small values of  $\alpha$  the effect of the Thomas-Fermi term becomes stronger: for  $\lambda = 10$ ,  $\alpha = 0.1$ ,  $\ell = 10$ , it lowers the result of (10)  $\rho_0 = 0.050$  to  $\rho_0 = 0.016$ , while the numerically obtained density for  $B = 20$  is near 0.04).

#### 4. Individual particles inside the bag

An interesting alternative way to define the Skyrme term in the lagrangian is to replace in (1) the topological current  $\rho^\mu$  (2) by the corresponding form in terms of the full vector field  $\Phi$

$$T^\mu = \frac{1}{8\pi} \epsilon^{\mu\nu\rho} \Phi \cdot (\partial_\nu \Phi \times \partial_\rho \Phi) = \Phi^3 \rho^\mu. \quad (13)$$

This introduces a factor  $\Phi^6$  into the Skyrme term. Due to this high power of  $\Phi$  there again is very little interference with the  $\Phi^4$  spontaneous symmetry-breaking mechanism of the  $\Phi$ -potential. However, apart from the fact that the length  $\ell$  now scales with  $f_0^{5/4}$  as  $f_0$  tends to zero, this alternative definition leads to a situation where, for fixed  $f_0$ , in spatial regions with small  $\Phi$  (i.e. in the interior of bags) the Zeeman term dominates the Skyrme term. This results in the stabilization of pointlike 'particles' inside the bag. Speaking again in the language of nuclear physics, one would say that the nucleons retain their individuality inside a bag which binds them together into a common  $B$ -nucleus.

Choosing as before the parameters  $\lambda = 10, \alpha = 1$  which favor the formation of deep bags, we find the energy minimum for  $B = 1$  at  $E_1 = 1.152$ . In the  $B = 2$  sector we find a minimum near  $E_2/2 = 0.943$  in which both charges share one common bag, but the individual charges stay apart from each other in two separate pockets. This '1+1'-configuration is shown on the left side of fig.6. There is, however, in this sector a lower minimum at  $E_2/2 = 0.663$  where the bag consists of only one single pocket which houses one doubly charged structure, i.e. the model favors the formation of pairs. This '2'-configuration is shown in the center plot of fig.6. For  $B = 3$  we find the expected '1+2'-configuration at  $E_3/3 = 0.697$ , a pair and one unit charge tightly bound in two

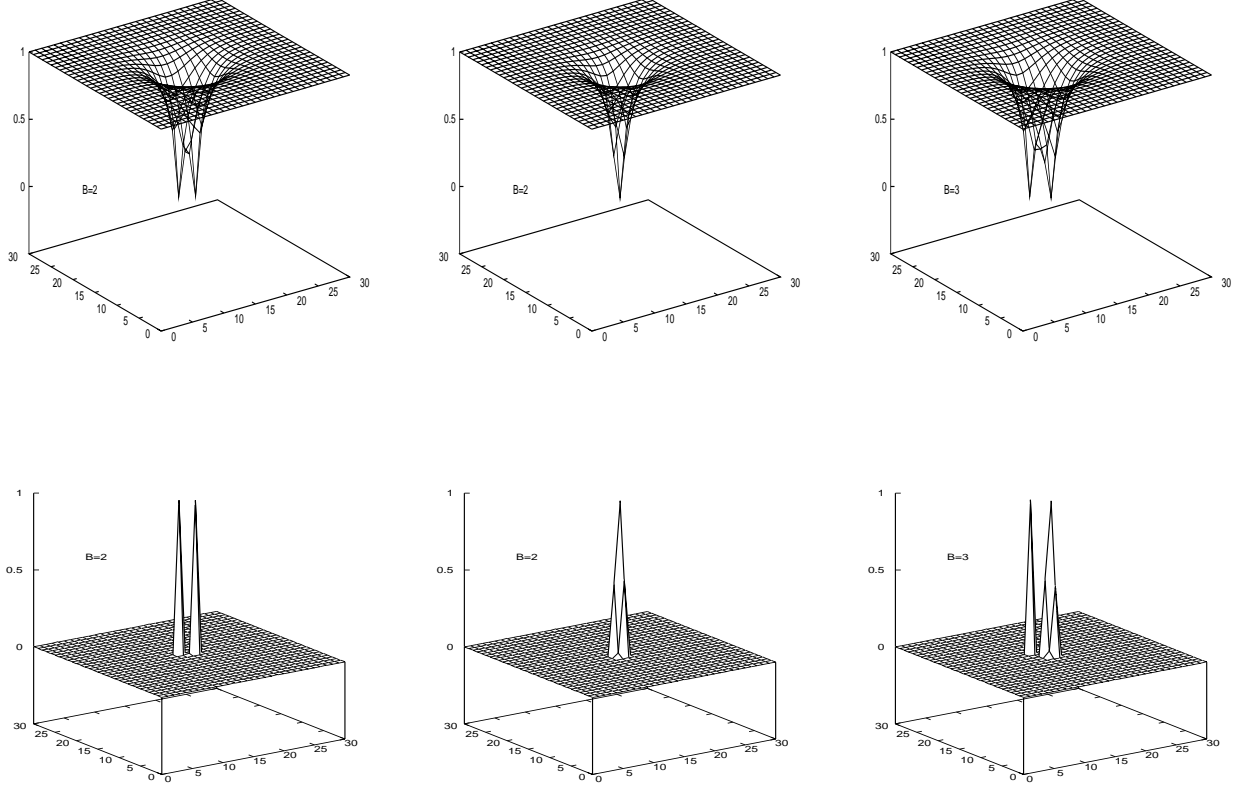


FIG. 6. 3D-view of the bags and corresponding winding densities for the '1+1' and '2' configurations observed in the  $B = 2$  sector (left and center) and the '1+2' configuration in the  $B = 3$  sector (right), if the Skyrme term in (1) is multiplied by  $\Phi^6$  as follows from the alternative definition (13). ( $\alpha = 1$ ,  $\lambda = 10$ ). The energies per unit charge are  $E_B/B = 0.943, 0.663, 0.697$ , respectively.

separate pockets of a common 3-bag, as shown on the right of fig.6. We do not find a minimal-energy '3'-configuration with a threefold charge sitting in one single pocket. The same situation is observed in the  $B = 4$  sector where again we do not find the single pocket with fourfold charge, but instead the '2+2' double pair separated in two pockets bound in one common 4-bag at  $E_4/4 = 0.543$ . So, it appears that the situation resembles very closely the pair formation as it was observed [4] for skyrmions for  $\lambda \rightarrow \infty$ . As in that case, the '1'- and '2'-configurations serve as building blocks which here are bound together in one deep bag. Due to this energetically favored pair formation the resulting 'nuclei' will be characterized by odd-even staggering of their binding energies for higher values of  $B$ . Practically, of course, the actual configuration which is finally reached through numerical relaxation depends on the starting set and the sequence of pseudo temperatures in the Metropolis cooling algorithm. As an example we present in fig.7 a result for  $B = 20$

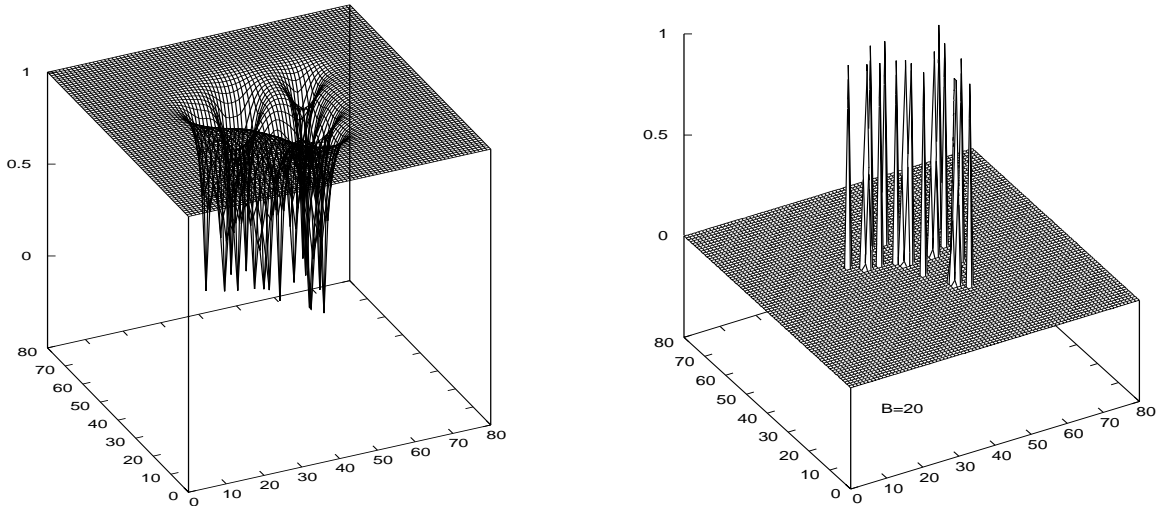


FIG. 7. 3D-view of the bag formed for  $B = 20$ , and the corresponding winding density  $\rho$ , if the Skyrme term in (1) is multiplied by  $\Phi^6$  as it follows from the alternative definition (13). The parameters ( $\alpha = 1$  and  $\lambda = 10$ ) are the same as in fig. 3b.

which shows a highly deformed bag with 15 pockets which houses a  $10 \times 1' + 5 \times 2'$  configuration. The calculation is done for the same parameter set as used in fig.3b, so a direct comparison shows nicely the contrast between pointlike particles and deconfined charge in the interior of one big bag.

## 5. Conclusion

We have investigated here the  $2D-O(3)$  model in a representation where the 3-vector field  $\Phi$  is split into the unit vector  $\hat{\Phi}$  and the modulus  $\Phi$ . This allows for the definition of a topological winding number  $B$ , and for the separation of the complete configuration space into distinct  $B$ -sectors. For small values of the  $\Phi^4$ -coupling strength  $\lambda$  the stable energy minima in these sectors are characterized by bag formation in the modulus field. In the standard cartesian representation of the linear  $O(3)$  model such configurations would be unstable towards decay into the trivial  $B = 0$  vacuum. Stabilized by  $B$ -conservation they exhibit a surprising variety of very appealing physics for multiply charged systems. For decreasing  $\lambda$  multi-skyrmions get bound into one common deep bag like nucleons get bound into one nucleus. Depending on the competition between Skyrme and Zeeman energy two opposite ways of distributing the topological charge inside the bag can be realized: Pointlike structures which keep the individuality of single nucleons (or doubly charged pairs) inside the nucleus, or a deconfined charge density spread uni-

formly throughout the interior of the bag. This latter case suggests a very close relation between the charge density  $\rho$  and the modulus field  $\Phi$  (which for sufficiently large  $B$  is  $\rho = \rho_0(1 - \Phi)$ ) and, correspondingly, an effective description through a density functional for  $\rho$ . This is a remarkable possibility because it gets rid of the angular fields altogether which form the basis for the definition of  $\rho$  ! It may be understood by the fact that in these configurations the angular fields are very rapidly fluctuating functions and a spatial coarse graining procedure will eliminate them. This is reminiscent of the fact that mean field or density functional methods for nucleons in nuclear physics work very well without keeping explicitly the dynamical pionic degrees of freedom. Similarly, in a recent analysis of soliton formation in the Nambu-Jona Lasinio model [6] it was found that without enforcing the chiral circle condition stable minima are characterized by vanishing pion field.

Of course, it will be most interesting to extend the present investigations to the  $3D$ - $O(4)$  model, where a lot of effort has gone into exploring  $3D$ - $SU(N_f)$  multi skyrmions [7] and their possible relevance for the structure of nuclei. Their spatial structure (which for large  $B$  looks somehow like buckey balls) has little in common with our naive picture of a nucleus. On the other hand it has been known for a long time [8] that inclusion of a scalar  $\sigma$  field is important for the attractive part of the skyrmion-skyrmion force. We should also rather expect a mechanism as described in sect.4 where the nucleons keep their individuality inside the bag. For dimensional reasons in the  $3D$ - $O(4)$  model there is no need for the Zeeman-type coupling to monitor the scale of the structures; it emerges directly from the competition between the second order (non linear  $\sigma$ ) term and the (fourth order) Skyrme term. Dominance of the Skyrme term (which may arise in the interior of the bag due to the factor  $\Phi^2$  multiplying the nonlinear  $\sigma$ -term, or due to increasing temperature which lowers the coefficient of the whole second order term) will again lead to deconfined baryon density inside the nucleus (or nuclear matter in the limit  $B \rightarrow \infty$ ). In any case, our present results strongly suggest that for a description of nuclei in terms of multiply charged solitons in chiral meson fields the constraint  $\Phi \equiv 1$  is too restrictive.

Creation of spatial regions with disoriented chiral condensate (DCC) in the course of a chiral symmetry-breaking transition has been studied in the framework of the linear  $O(4)$  model in trivial topology [9]. It will be of interest to investigate how the existence of the various phase regions explored here for the simple  $2D$ - $O(3)$  model may affect the conclusions to be drawn for defect and DCC formation in case of the chiral phase transition [10].

Finally we should note that all results reported here have been obtained through numerical relaxation of a lattice functional through some Metropolis cooling algorithm. This naturally poses the question about the continuum limit of these results. Apparently, (as is directly obvious from fig.4), this is not a trivial matter because  $B$  conservation relies on the removal of one single point ( $\Phi = 0$ ) from the manifold on which the model is defined. But this is the usual mathematical difficulty encountered in the transition from a granular to a continuous density distribution. And, again as usual, we have no proof that we really have obtained the lowest energy minima in the respective  $B$ -sectors.

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